Theorem Fig

$$\overline{az+bz^2+cz^3+dz^4+ez^5+fz^6+gz^7 \ldots}$$

$$=a^m z^m + \frac{m}{1}a^{m-1}bz^{m+1} + \frac{m}{1}\times\frac{m-1}{2}a^{m-2}b^2z^{m+2} + \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}a^{m-3} \ldots$$

$$+\frac{m}{1}a^{m-1}c \qquad +\frac{m}{1}\times\frac{m-1}{1}a^{m-2} \ldots$$

$$+\frac{m}{1}a^{m-1} \ldots$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}a^{m-5}b^5z^{m+5}$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{1}a^{m-4}b^3c$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}a^{m-3}b^2d$$

$$+\frac{m}{1}\times\frac{m-1}{1}\times\frac{m-2}{2}a^{m-3}bc^2$$

$$+\frac{m}{1}\times\frac{m-1}{1}a^{m-2}be$$

$$+\frac{m}{1}\times\frac{m-1}{1}a^{m-2}cd$$

$$+\frac{m}{1}a^{m-1}f$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}\times \ldots$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times \ldots$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times \ldots$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}\times \ldots$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times \ldots$$

$$+\frac{m}{1}\times\frac{m-1}{1}\times \ldots$$

$$+\frac{m}{1}\times \ldots$$

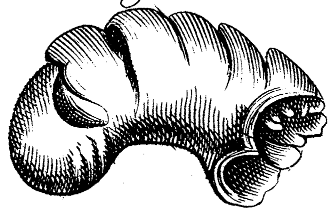
$$+\frac{m}{1}\times\frac{m-1}{2}\times \ldots$$

$$+\frac{m}{1}\times \ldots$$

$$+\frac{m}{1}\times \ldots$$

$$+ \ldots$$

Fig: 1



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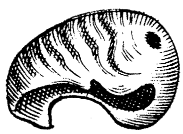

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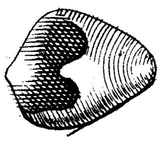

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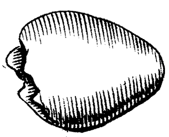


$$\overline{gz^7+hz^8+iz^9\ \&c}\,\rceil^{m} =$$

$a^{m-3}b^3z^{m+3}$ $+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}a^{m-4}b^4z^{m+4}$

$a^{m-2}bc$ $+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}a^{m-3}b^2c$

$a^{m-1}d$ $+\frac{m}{1}\times\frac{m-1}{1}a^{m-2}bd$

$+\frac{m}{1}\times\frac{m-1}{2}a^{m-2}c^2$

$+\frac{m}{1}a^{m-1}e$

$\frac{m-4}{5}\times\frac{m-5}{6}a^{m-6}b^6z^{m+6}\ \&c$

$\frac{m-3}{4}\times\frac{m-4}{1}a^{m-5}b^4c$

$\frac{m-2}{3}\times\frac{m-3}{1}a^{m-4}b^3d$

$\frac{m-2}{1}\times\frac{m-3}{2}a^{m-4}b^2c^2$

$\frac{m-1}{2}\times\frac{m-2}{1}a^{m-3}b^2e$

$\frac{m-1}{1}\times\frac{m-2}{1}a^{m-3}bcd$

$\frac{m}{1}\times\frac{m-1}{1}a^{m-2}bf$

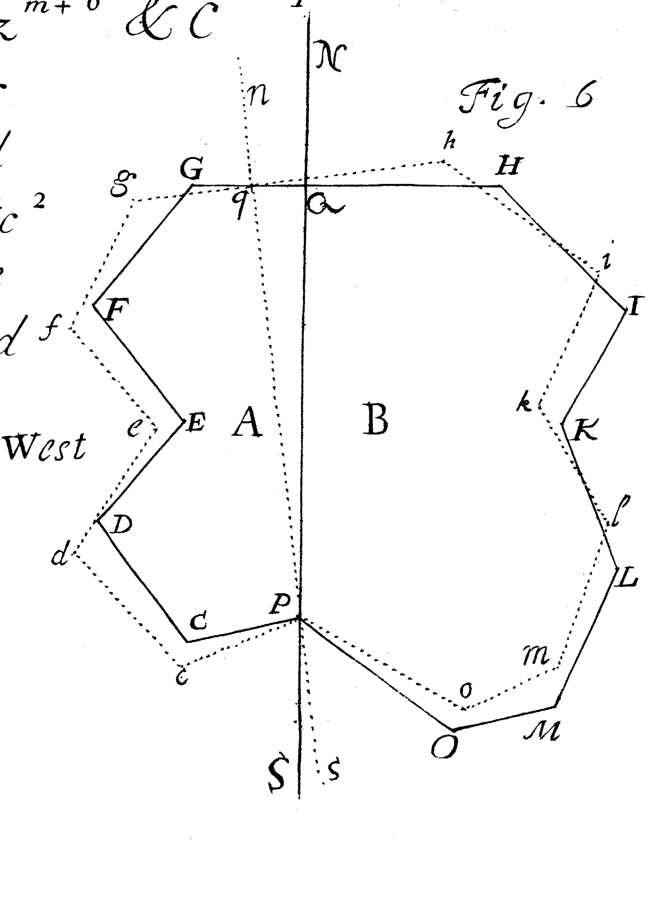
$\frac{m-1}{2}\times\frac{m-2}{3}a^{m-3}c^3$

$\frac{m}{1}\times\frac{m-1}{1}a^{m-2}ce$

$\frac{m}{1}\times\frac{m-1}{2}a^{m-2}d^2$

$+\frac{m}{1}a^{m-1}g$

Fig. 6



ſwim ſomewhat deep in the Water, ſometimes are catch'd, though not often. The Seamen have reached them with a Fiſgig, a kind of barbed Iron, at the end of a Pole tyed faſt to a Rope, and have made good chear with them. But this is only my Conjecture, with which I end my Journal. *Deo Servatori Laus.*

III. *A Method of Raiſing an infinite Multinomial to any given Power, or Extracting any given Root of the ſame. By Mr.* Ab. De Moivre.

'TIS about two Years ſince, that conſidering Mr. *Newton*'s Theorem for Raiſing a Binomial to any given Power, or Extracting any Root of the ſame; I enquired, whether what he had done for a Binomial, could not be done for an infinite Multinomial. I ſoon found the thing was poſſible, and effected it, as you may ſee in the following Paper; I deſign in a little time to ſhew the Uſes it may be applied to: In the mean while, thoſe that are already verſ'd in the Doctrine of Infinite Series, and have ſeen what Applications Mr. *Newton* has made of his Theorem, may of themſelves derive ſeveral Uſes from this.

I ſuppoſe that the Infinite Number Multinomial is $az + bzz + cz^3 + dz^4 + ez^5$ &c. m is the Index of the Power, to which this Multinomial ought to be Rais'd, or if you will, 'tis the Index of the Root which is to be Extracted: I ſay that this Power or Root of the Multinomial, is ſuch a Series as I have expreſt.

For the Underſtanding of it, it is only neceſſary to conſider all the Terms by which the ſame Power of z is Multiplied; in order thereto I diſtinguiſh two things in each of theſe Terms; 1° The Product of certain

Powers

Powers of the quantities, a, b, c, d, &c. 2° the *Unciæ* (as *Oughtred* calls 'em) prefixt to these Products. To find all the Products belonging to the same Power of z, to that Product, for instance, whose Index is $m+r$ (where r may denote any integer Number) I divide these Products into several *Classes*; those which immediately after some certain Power of a (by which all these Products begin) have b, I call *Products* of the 1rst *Classis*; For *Example* $a^{m-4}b^3e$ is a Product of the 1rst *Classis*, because b immediately follows a^{m-4}; those which immediately after some *Power* of a have c, I call Products of the 2d *Classis*, so $a^{m-3}ccd$ is a Product of the 2d Classis; Those which immediately after some Power of a have d, I call Products of the 3d Classis, and so of the rest.

This being done, I Multiply all the Products belonging to z^{m+r-1} (which precedes immediately z^{m+r}) by b and Divide 'em all by a; 2° I Multiply by c and Divide by a, all the Products belonging to z^{m+r-2}, Except those of the 1rst Classis; 3° I Multiply by d and Divide by a all the Products belonging to z^{m+r-3}, Except those of the 1rst and 2d Classis, 4° I Multiply by e and Divide by a all the Terms belonging to z^{m+r-4}, Except those of the 1rst, 2d and 3d *Classis*, and so on, till I meet twice with the same term. Lastly, I add to all these Terms the Product of a^{m-1} into the Letter whose *Exponent* is $r+1$.

Here I must take notice that by the Exponent of a Letter, I mean the number which expresses what Place the Letter has in the *Alphabet*, so 3 is the Exponent of the Letter c because the Letter c is the 3d in the *Alphabet*.

It is evident that by this Rule, you may easily find all the Products belonging to the several Powers of z, if you have but the Product belonging to z^m, *viz.* a^m.

To

To find the *Unciæ* which ought to be prefixt to every Product, I consider the Sum of Units contained in the Indices of the Letters which compose it (the Index of a excepted) I write as many Terms of the Series $m\times m-1\times m-2\times m-3$, *&c.* as there are *Units* in the Sum of these Indices, this Series is to be the Numerator of a Fraction, whose Denominator is the Product of the several Series $1\times2\times3\times4\times5$, *&c.* $1\times2\times3\times4\times5$, *&c.* $1\times2\times3\times4\times5\times6$, *&c.* the 1[ft] of which contains as many Terms as there are *Units* in the Index of b, the 2[dl] as many as there are *Units* in the Index of c, the 3[d] as many as there are *Units* in the Index of d, the 4[th] as many as there are *Units* in the Index of e, *&c.*

Demonstration.

To raise the Series $az+bzz+cz^3+dz^4$, *&c.* to any Power whatsoever, write so many Series equal to it as these are *Units* in the Index of the Power demanded. Now it is evident that when these Series are so Multiplied, there are several Products in which there is the same Power of z, thus if the Series $az+bzz+cz_3+dz^4$ *&c.* is rais'd to its Cube, you have the Products b^3z^6, $abcz^6$, $aadz^6$, in which you find the same Power z^6 Therefore let us consider what is the Condition that can make some Products to contain the same Power of z, the first thing that will appear in relation to it, is that in any Product whatsoever, the Index of z is the Sum of the particular Indices of z in the Multiplying Terms (this follows from the the nature of Indices) thus b^3z^6 is the Product of bz^2, bz^2, bz^2, and the Sum of the Indices in the Multiplying Terms, is $2+2+2=6$; $abcz^6$ in the Product of az, bzz, cz^3, and the Sum of them Indices of z in the Multiplying Terms is $1+2+3=6$ $aadz^6$ is the Product of az, az, dz^4, and the Sum of the

Indices of z in the Multiplying Terms is $1+1+4=6$; the next thing that appears is, that the Index of z in the Multiplying Terms is the ſame with the Exponent of the Latter to which z is joyn'd, from which two Conſiderations it follows that, *To have all the Products belonging to a certain Power of z, you muſt find all the Products where the Sum of the Exponents of the Letters which compoſe 'em ſhall always be the ſame with the Index of that Power.* Now this is the method I uſe to find eaſily all the Products belonging to the ſame *Power* of z, Let $m+r$ be the Index of that Power, I conſider that the Sum of the Exponents of the Letters which compoſe theſe Products muſt exceed by 1 thoſe which belong to m^{m+r-1}, now becauſe the exceſs of the Exponent of the Letter b above the Exponent of the Letter a, is 1, it follows that if each of the Products belonging to z^{m+r-1} is Multipli'd by b, and Diuided by a, you will have Products the Sum of whoſe Exponents will be $m+r$; Likewiſe the Sum of the Exponents of the Letters which compoſe the Products belonging to z^{m+r} exceeds by 2 the Sum of the Exponents of the Letters which compoſe the Products belonging to z^{m+r-2}; Now becauſe the Exponent of the Letter a is leſs by 2 than the Exponent of the letter c, it follows, that if each Product belonging to z^{m+r-2} is Multipli'd by c and Divided by a, you will have other Products the Sum of whoſe Exponents is ſtill $m+r$; Now if all the Products belonging to z^{m+r-2} were Multiplied by c and Divided by a, you would have ſome Products that would be the ſame as ſome of thoſe found before, therefore you muſt except out of 'em thoſe that I have call'd Products of the 1^rſt^ *Claſſis*; what I have ſaid ſhows why all the Products belonging to z^{m+r-3}, except thoſe of the 1^rſt^ and 2^d^ *Claſſis* muſt be Multipli'd by d and Divided by a: Laſtly, you ſee the Reaſon why to all theſe Products is added

added the Product of a^{m-1} by the Letter whose Exponent is $r+1$; 'Tis because the Sum of the *Exponents* is still $m+r$.

As for what relates to the *Unciæ*; observe that when you Multiply $az+bzz+cz^3+dz^4$, *&c.* by $az+bzz+cz^3$ dz^4, *&c*, each Letter *a*, *b*, *c*, *d*, *&c.* of the 2[d] Series is Multipli'd by each of the Letters *a*, *b*, *c*, *d*, *&c.* of the 1[rst] Series; Thus the Letter *a* of the 2[d] Series is Multipli'd by the letter *b* of the 1[rst], and the letter *b* of he 2[d] Series is Multipli'd by the Letter *a* of the 1[rst]; therefore you have the 2 Planes, *ab*, *ab* or $2ab$; for the same reason you have $2ac$, $2ad$, &c. Therefore you must prefix to each Plane of those that compose the Square of the Infinite Series $az+bzz+cz^3$, &c. the Number which expresses how many Ways the Letters of each Plane may be changed; Likewise if you Multiply the Product of the two preceeding Series by $az+bzz+cz^3$, &c. each Letter *a*, *b*, *c*, *d*, of the 3[d] Series is Multiplied by each of the Planes form'd by the Product of the 1[rst] and 2[d] Series; Thus the Letter *a* is Multiplied by the Planes *bc* and *cb*; the Letter *b* is Multiplied by *ac* and *ca*; the Letter *c* is Multiplied by *ab* and *ba*; therefore you have the 6 Solids, *abc*, *acb*, *bac*, *bca*, *cab*, *cba*, or $6abc$; Therefore you must prefix to each Solid whereof the Cube of the Infinite Series is compos'd, the Number which expresses how many ways the Letters of each Solid may be Changed. And generally, *You must prefix to any Product whereof any Power of the Infinite Series* $az+bzz+cz^3$, &c. *is compos'd the Number which expresses how many ways the Letter of each Product may be changed.*

Now to find how many ways the Letters of any Product, for instances $a^{m-n}b^h c^p d^r$ may be changed; this is the Rule which is commonly given: write as many terms of the Series $1\times2\times3\times4\times5$, *&c.* as there are Units in the Sum of the Indices, *viz.* $m-n+h+p+r$, let this Series be the Numerator of a Fraction whose Denominator shall be the Product of the Series, $1 \times 2 \times 3 \times 4 \times 5$, &c. $1 \times 2 \times 3 \times 4 \times 5$, &c. $1 \times 2 \times 3 \times 4 \times 5 \times 6$, &c $1 \times 2 \times 3 \times 4 \times 5$, &c. whereof the 1rst is to contain as many Terms, as there Units in the 1rst Index $m-n$; the 2d as many as there are Units in the 2d Index h; the 3d as many as there are Units in the 3d Index p; the 4th as many as there are Units in the 4th Indexs r. But the Numerator and Denominator of this Fraction have a common Divisor, *viz.* the Series $1 \times 2 \times 3 \times 4 \times 5$, &c. continued to so many Terms as there are Units in the 1rst Index $m-n$; therefore let both this Numerator and Denominator be Divided by this common Divisior, then this new Numerator will begin with $m-n+1$, whereas t'other began with 1, and will contain so many Terms as there are Units in $h+p+r$, that is so many as there are Units in the Sum of all the Indices, excepting the 1rst; as for the new Denominator, it will be the Product of 3 Series only, that is of so many as there Indices, excepting the 1rst. But if it happens withal, that n be equal to $h+p+r$ as it always happens in our *Theorem*, then the Numerator beginning by $m-n+1$, and being continued to so many Terms as there are Units in $h+p+r$ or n, the last Term will be m necessarily, so if you invert the Series and make that the first Term which was the Last, the Numerator will be $m \times m-1 \times m-2 \times m-3$, &c. conntinued to so many Terms as there are Units in the Sum of the Indices of each Product, excepting the 1rst Index. There remains but one thing to demonstrate which is, that, what I have said of Powers whose Index is an Integer, may be adapted to Roots, or Powers whose Index is a Fraction; but it appears at first sight why it should be so: for, the same Reason which makes me consider Roots under the Notion of Powers, will make me conclude, that

that whatever is ſaid of one may be ſaid of t'other. However I think to give ſometime a more formal Demonſtration of it.

See the Theorem *Fig.* 5.

IV. *A Demonſtration of an Error committed by common Surveyors in comparing of Surveys taken at long Intervals of Time ariſing from the Variation of the Magnetick Needle*, *by* William Molyneux *Eſq*; *F. R. S.*

THE Variation of the Magnetick Needle is ſo commonly known, that I need not inſiſt much on the Explication thereof, 'tis certain that the true Solar Meridian, and the Meridian ſhewn by a Needle, agree but in very few places of the World; and this too, but for a little time (if a Moment) together. The Difference between the true Meridian and Magnetick Meridian perpetually varying and changing in all Places and at all Times; ſometimes to the Eaſtward and ſometimes to the Weſtward.

On which account 'tis impoſſible to compare two Surveys of the ſame place, taken at diſtant times, by Magnetick Inſtruments (ſuch as the *Circumferentor*, by which the *Down Survey*, or Sir *William Petty*'s Survey of *Ireland* was taken) without due allowance be made for this Variation. To which purpoſe we ought to know the Difference between the Magnetick Meridian and true Meridian at that time of the Down Survey, and the ſaid Difference at the time, when we make a New Survey to compare with the Down Survey.

Theorem Fig. 5. Transact. No 230.

$$\overline{az + bz^2 + cz^3 + dz^4 + ez^5 + fz^6 + gz^7 + hz^8 + iz^9 \ \&c}\Big|^{m} =$$

$$= a^m z^m + \frac{m}{1} a^{m-1} b z^{m+1}$$

$$+ \frac{m}{1}\times\frac{m-1}{2} a^{m-2} b^2 z^{m+2} + \frac{m}{1} a^{m-1} c$$

$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3} a^{m-3} b^3 z^{m+3} + \frac{m}{1}\times\frac{m-1}{1} a^{m-2} bc + \frac{m}{1} a^{m-1} d$$

$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4} a^{m-4} b^4 z^{m+4} + \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1} a^{m-3} b^2 c + \frac{m}{1}\times\frac{m-1}{1} a^{m-2} bd + \frac{m}{1}\times\frac{m-1}{2} a^{m-2} c^2 + \frac{m}{1} a^{m-1} e$$

$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5} a^{m-5} b^5 z^{m+5} + \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{1} a^{m-4} b^3 c + \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1} a^{m-3} b^2 d + \frac{m}{1}\times\frac{m-1}{1}\times\frac{m-2}{2} a^{m-3} bc^2 + \frac{m}{1}\times\frac{m-1}{1} a^{m-2} be + \frac{m}{1}\times\frac{m-1}{1} a^{m-2} cd + \frac{m}{1} a^{m-1} f$$

$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}\times\frac{m-5}{6} a^{m-6} b^6 z^{m+6} \ \&c$$

$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{1} a^{m-5} b^4 c$$

$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{1} a^{m-4} b^3 d$$

$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}\times\frac{m-3}{2} a^{m-4} b^2 c^2$$

$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1} a^{m-3} b^2 e$$

$$+ \frac{m}{1}\times\frac{m-1}{1}\times\frac{m-2}{1} a^{m-3} bcd$$

$$+ \frac{m}{1}\times\frac{m-1}{1} a^{m-2} bf$$

$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3} a^{m-3} c^3$$

$$+ \frac{m}{1}\times\frac{m-1}{1} a^{m-2} ce$$

$$+ \frac{m}{1}\times\frac{m-1}{2} a^{m-2} d^2$$

$$+ \frac{m}{1} a^{m-1} g$$

Fig. 6

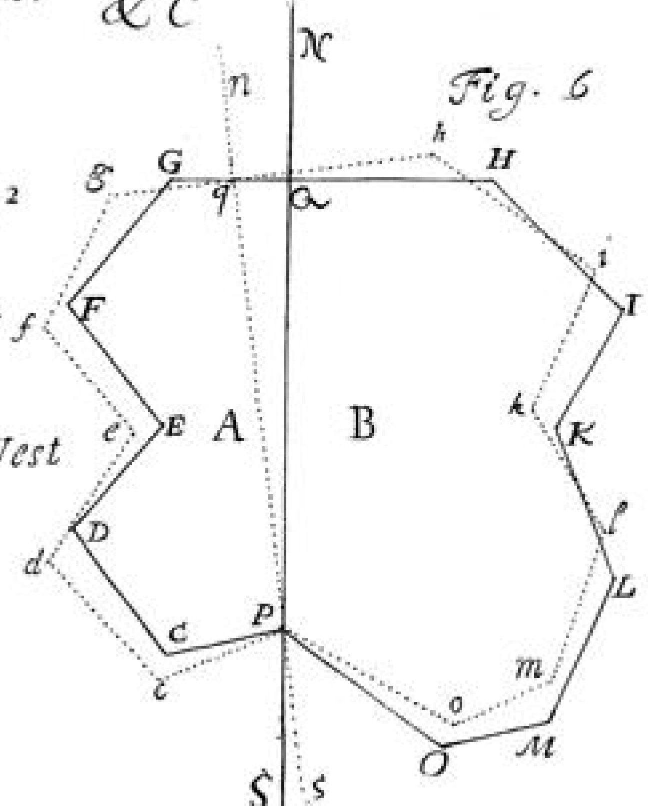


Fig: 1

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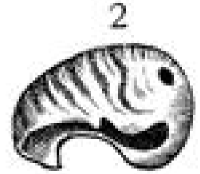

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